# AN EVALUATION OF A MODIFIED PRODUCT SIZE DISTRIBUTION MODEL BASED ON T-FAMILY CURVES FOR THREE DIFFERENT CRUSHERS 

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#### Abstract

Crushing is a process which is widely used in mineral processing plants, cement factories, aggregates plants and some other industrial plants. Specific fracture energy of the particles is not the only fundamental property that is important: the particle strength also plays a significant role in determining the overall comminution properties of the material. In the drop weight test, a known mass falls through a given height onto a single particle providing an event that allows characterization of the ore under impact breakage. It is known that there are many difficulties and problems in the drop weight and twin pendulum test methods such as being laborious, requiring long test time and requiring a special apparatus. In this study, breakage behavior of slag in different laboratory crushers was investigated. A new size distribution model equation was developed by a t-family value evaluation approach, and the validity of equation was tested.


Keywords: crusher, drop weight tests, t-family, size distribution

## Introduction

Comminution concerns the breakage of brittle particles under conditions of applied compressive stress. The nature of the failure mechanisms is governed by material properties of the particulate material and by the nature of the stress field around and within individual particles. The response of the particulate material to the stress field is largely elastic but significant non-elastic behavior occurs, particularly at the tips of growing cracks where large quantities of energy are dissipated when the criteria for fracture are met. The dissipation at the crack tip of the stored elastic energy in the particle turns out to be of critical interest in industrial comminution machines where energy efficiency is of major consequence because of its economic importance. Industrial comminution processes are typically inefficient in their use of energy in the sense that considerably more energy is consumed by the operating equipment than is
actually required to break the particles. In spite of the importance of this observation, it has not been possible to calculate precisely how much energy is actually required.

Crushing is a process which is widely used in mineral processing plants, cement factories, aggregates plants and some other industrial plants. Especially, the demand of crushed stone aggregates has increased from day to day, because of increasing expansion of highway and other construction works and decreasing natural aggregate resources in the world. The crushability of rocks depends mainly on the crusher type and properties, and geological characteristics of rocks. Physico-mechanical properties, mineralogical and textural features, structural properties such as micro cracks, cleavage planes and schistosity are the geological characteristics affecting the rock crushability (Toraman et al., 2010)

Specific fracture energy of the particles is not the only fundamental property that is important. The particle strength also plays a significant role in determining the overall comminution properties of the material. A particle will be broken only if it is stressed beyond its strength which is determined by the intrinsic properties of the material, the presence of micro flaws which act as stress raisers when the particle is under load and the state of stress that is experienced by the particle.

In the drop weight test (Fig. 1) a known mass falls through a given height onto a single particle providing an event that allows characterization of the ore under impact breakage. Although, the drop weight test has advantages in terms of statistical reliability and the potential use of the data from the analysis, it has a number of disadvantages, including necessity of a special apparatus, tiring and particularly the length of time taken to carry out a test. For each drop weight test, 15 samples are tested in five size fractions at three levels of energy input (Tavares, 1999, Tavares et al., 2007, Kingman et al., 2004, Genç et al., 2009).

Narayanan (1986) used a novel procedure for estimation of breakage distribution functions of ores from the t-family of curves. In this method, the product size distribution can be represented by a family of curves using marker points on the size distribution defined as the percentage passing $(t)$ at a fraction of the parent particle size. Thus, $t_{2}$ is the percentage passing an aperture of half the size of the parent particle size, $t_{4}$ is one quarter and $t_{10}$ is one-tenth of parent particle size.

Narayanan and Whiten (1988) have proposed empirical equations for relating the reference curve data $t_{10}$ with the impact energy.

The $t_{10}$ value is related to the specific comminution energy given by Equation (1):

$$
\begin{equation*}
t_{10}=A\left(1-\mathrm{e}^{-b E c s}\right) \tag{1}
\end{equation*}
$$

where: $t_{10}$ - percentage passing $1 / 10^{\text {th }}$ of the initial mean size,
Ecs - specific comminution energy ( $\mathrm{kWh} / \mathrm{kg}$ )
$A, b$ - ore impact breakage parameters.
The $t_{n}$ versus $t_{10}$ relationships can then be used to predict the product size distributions at different crushing times (Sand and Subasinghe, 2004).


Fig. 1. Drop-weight test apparatus
It is known that there are difficulties and problems in drop weight and twin pendulum test methods such as being laborious, requiring long test time and requiring a special apparatus. In this study, the breakage behaviors of the volcanic slag in the different laboratory crushers were investigated. A new size distribution model equation was developed by t-family value evaluation approach and validity of the specific energy equation was tested.

## Materials and methods

The volcanic slag sample taken from region of Manisa (Turkey) was used as the experimental materials. The chemical properties of the sample were presented in Table 1.

Table 1. Chemical composition of volcanic slag samples used in experiments

| Oxides | CaO | $\mathrm{SiO}_{2}$ | $\mathrm{Al}_{2} \mathrm{O}_{3}$ | $\mathrm{Fe}_{2} \mathrm{O}_{3}$ | MgO | $\mathrm{SO}_{3}$ | $\mathrm{Na}_{2} \mathrm{O}$ | $\mathrm{K}_{2} \mathrm{O}$ | LOI |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | ---: |
| $(\%)$ | 8.73 | 45.74 | 15.60 | 10.64 | 5.98 | 0.040 | 5.40 | 4.98 | 1.70 |

### 2.1. Experimental

About one kg of samples of four mono-size fractions $(-30+20,-20+15,-15+10$, $10+6.7 \mathrm{~mm}$ ) was prepared by screening for determination of the $t$-family curves. The laboratory scale jaw, roller and hammer crushers (Fig. 2), driven by a $1.10-2.00$ 11.00 kW motor, respectively, was used in the experiments. Samples were crushed with the laboratory scale crushers, and then the samples were sieved to product size analysis. The modified results of the $t$-family curves versus mean particle size fraction for different crushers were shown in Figs. 3-5.


Fig. 2. Jaw, roller and hammer crushers used in the experiments


Fig. 3. $t_{n}$ versus mean size fraction for jaw crusher


Fig. 4. $t_{n}$ versus mean size fraction for roller crusher


Fig. 5. $t_{n}$ versus mean size fraction for hammer crusher

## Proposed size distribution model equation

King (2002) described a method of presenting product size distributions obtained from the drop weight tests. It is based on the observations made by Narayanan and Whiten (1983) that the cumulative fractions of products passing $1 / \mathrm{n}^{\text {th }}$ of the mean size, denoted as $t_{n}$, is related to that passing one-tenth of the parent size denoted by $t_{10}$. It was also reported that this relationship was applicable to different ore types tested under different impact loading conditions.

In this study, a different size distribution relationship has been found for $t_{n}$ values of crushing products obtained from different laboratory scale crushers. The following equation is suggested to predict the cumulative percentage passing $\left(t_{n}\right)$, depending on the crushing engine power ( $C E P$ ) and feed particle size $(X)$ before crushing:

$$
\begin{equation*}
t_{n}=\left(\frac{56.12}{n^{1.168 C E P^{-1.274}}}\right) X^{0.30} \tag{2}
\end{equation*}
$$

and the calculated results obtained by Equation (2) were compared with experimental values obtained from results of Fig. 6. Equation (2) satisfied the experimental values in a wide range of feed size. Equation (2), including a high determination coefficient ( $r^{2}=0.79$ ), is useful, especially when evaluating the particle size distribution in actual operation. Thus, variation of the cumulative percentage passing $\left(t_{n}\right)$ with crushing engine power ( $C E P$ ) and feed particle size ( $X$ ) was empirically described by Eq. (2).


Fig. 6. Comparison of experimental and calculated $t_{n}$ value for different crushers

## Results and discussion

As commonly known, the type of crusher plays an important role in comminution while electrical engine power is important in crusher selection. The modified t-family curves can be used to describe the product size distribution from single-particle
breakage tests. The cumulative percentages passing $t_{2}, t_{4}, t_{10}, t_{25}, t_{50}$ and $t_{75}$ are determined by a linear interpolation for the breakage products passing various sieves. However, there are problems by drop weight and twin pendulum tests because of being laborious, requiring long test time and a special apparatus.

Evaluation of single particle breakage data through a single parameter is useful for understanding breakage characteristics, namely the size and input energy level dependency. Breakage characteristics of different size fractions can be examined at various impact energies expressed in $\mathrm{kWh} / \mathrm{kg}$ or joules/kg by determining the product size distribution of broken particle at any energy level (Genç et al., 2004).

Obtaining the modified $t$-family curves using the twin pendulum device and drop weight apparatus is time consuming and difficult. Therefore, a set of $t$-curves were calculated from different laboratory scale crushers, and a new model was developed. There are only few studies on the effects of work speed, lining design, crusher design and feed dose of materials in crushers, which are widely used in power, cement and mineral processing plants. For this purpose, similar mathematical models can be developed for different work parameters for various materials and various crusher types (Umucu et al., 2013).

## Conclusions

The individual particle fracture strengths of materials based on fracture size can benefit from drop weight test results. The models of crushing can be used to results of such tests. An important problem in terms of crushing modeling studies is sizedependent breakage behavior of materials. The breakage data of a single particle broken at an appropriate specific comminution energy level are commonly determined as breakage distributions of the crushing feed materials. In the comminution, the models should define the breakage distribution of each size fraction in the same material.

The effects of feed particle size ( $X$ ) and electrical crushing engine power on product size distribution were investigated. The proposed model equation which incorporates the self-similar breakage behavior during laboratory comminution could be used as an alternative to the drop weight and twin pendulum tests, as its particle size distributions could be defined more readily and reliably.

The drop weight test allows determining the breakage distribution of materials. The result of comminution, breakage distribution and energy consumed cannot be obtained without laboratory-scale apparatus as the drop weight tests. The experimental work should be done on an industrial scale. The $t$-family curves can characterize comminution products of any crusher. The model proposed in this study can be improved for crushers. In this study, the modified t-family model shows easy determination of the product size distribution of any material.

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